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APPLICATION NO.	FILING DATE	FIRST NAMED INVENTOR	ATTORNEY DOCKET NO.	CONFIRMATION NO.
09/696,432	10/25/2000	Dumitru Mihai Ionescu	NC17502	2501

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EXAMINER	
LIU, SHUWANG	

ART UNIT	PAPER NUMBER
2634	

DATE MAILED: 12/12/2003 14

Please find below and/or attached an Office communication concerning this application or proceeding.

PTO-90C (Rev. 10/03)

H

Advisory Action	**Application No.**	**Applicant(s)**	
	09/696,432	IONESCU, DUMITRU MIHAI	
	Examiner	**Art Unit**	
	Shuwang Liu	2634	

--The MAILING DATE of this communication appears on the cover sheet with the correspondence address --

THE REPLY FILED FAILS TO PLACE THIS APPLICATION IN CONDITION FOR ALLOWANCE.
Therefore, further action by the applicant is required to avoid abandonment of this application. A proper reply to a final rejection under 37 CFR 1.113 may only be either: (1) a timely filed amendment which places the application in condition for allowance; (2) a timely filed Notice of Appeal (with appeal fee); or (3) a timely filed Request for Continued Examination (RCE) in compliance with 37 CFR 1.114.

PERIOD FOR REPLY [check either a) or b)]

a) ☐ The period for reply expires ______months from the mailing date of the final rejection.

b) ☒ The period for reply expires on: (1) the mailing date of this Advisory Action, or (2) the date set forth in the final rejection, whichever is later. In no event, however, will the statutory period for reply expire later than SIX MONTHS from the mailing date of the final rejection.
ONLY CHECK THIS BOX WHEN THE FIRST REPLY WAS FILED WITHIN TWO MONTHS OF THE FINAL REJECTION. See MPEP 706.07(f).

Extensions of time may be obtained under 37 CFR 1.136(a). The date on which the petition under 37 CFR 1.136(a) and the appropriate extension fee have been filed is the date for purposes of determining the period of extension and the corresponding amount of the fee. The appropriate extension fee under 37 CFR 1.17(a) is calculated from: (1) the expiration date of the shortened statutory period for reply originally set in the final Office action; or (2) as set forth in (b) above, if checked. Any reply received by the Office later than three months after the mailing date of the final rejection, even if timely filed, may reduce any earned patent term adjustment. See 37 CFR 1.704(b).

1.☐ A Notice of Appeal was filed on _____. Appellant's Brief must be filed within the period set forth in 37 CFR 1.192(a), or any extension thereof (37 CFR 1.191(d)), to avoid dismissal of the appeal.

2.☐ The proposed amendment(s) will not be entered because:

(a) ☐ they raise new issues that would require further consideration and/or search (see NOTE below);

(b) ☐ they raise the issue of new matter (see Note below);

(c) ☐ they are not deemed to place the application in better form for appeal by materially reducing or simplifying the issues for appeal; and/or

(d) ☐ they present additional claims without canceling a corresponding number of finally rejected claims.

NOTE: _____.

3.☐ Applicant's reply has overcome the following rejection(s): _____.

4.☐ Newly proposed or amended claim(s) _____ would be allowable if submitted in a separate, timely filed amendment canceling the non-allowable claim(s).

5.☒ The a)☐ affidavit, b)☐ exhibit, or c)☒ request for reconsideration has been considered but does NOT place the application in condition for allowance because: *See Continuation Sheet*.

6.☐ The affidavit or exhibit will NOT be considered because it is not directed SOLELY to issues which were newly raised by the Examiner in the final rejection.

7.☐ For purposes of Appeal, the proposed amendment(s) a)☐ will not be entered or b)☐ will be entered and an explanation of how the new or amended claims would be rejected is provided below or appended.

The status of the claim(s) is (or will be) as follows:

Claim(s) allowed: _____.

Claim(s) objected to: _____.

Claim(s) rejected: _____.

Claim(s) withdrawn from consideration: _____.

8.☐ The drawing correction filed on _____ is a)☐ approved or b)☐ disapproved by the Examiner.

9.☐ Note the attached Information Disclosure Statement(s)(PTO-1449) Paper No(s). _____.

10.☐ Other: _____

Shuwang Liu
Primary Examiner
Art Unit: 2634

Continuation of 5. does NOT place the application in condition for allowance because: The arguments offered by the Applicant have been addressed sufficiently in the Examiner's office action and the Examiner's position remains unchanged (see attachement) .

Attachments

Response to Arguments

1. Applicant's arguments filed 0n 11/26/03 have been fully considered but they are not persuasive.

The Examiner has thoroughly reviewed Applicant's arguments but firmly believes that the cited reference reasonably and properly meet the claimed limitation as rejected.

(1) regarding the rejection under the section112, 1st paragraph:

Applicant's argument – "Specific note is made of the equation set forth on page 16, line 2 as such equation states mathematically the recitation of the difference matrix multiplied together with a Hermetian matrix thereof being proportional to an identity matrix (I)."

Examiner's response – The equation set forth on page 16, line 2, is $\mathbf{D^H_{ec}D_{ec} = (tr(D^H_{ec}D_{ec})/L_t)I_l)}$, where D_{ec} is the difference matrix, D^H_{ec} is adjoint matrix (H denotes conjugated transposition), $tr(D^H_{ec}D_{ec})$ is the Euclidean distance, and I is assumed as the identity matrix. Which one is the Hermetian matrix? The specification (the equation on page 16, line 2) fails to describe a difference matrix multiplied together with a Hermetian matrix thereof being proportional to an identity matrix.

(2) regarding the rejection under section 102 (e):

Applicant's argument – "There is no disclosure of the difference matrix multiplied together with a Hermetian matrix thereof being proportional to an identity matrix."

Examiner's response –As agreed by the applicant, the equation 8 of Calderbank is a difference matrix B (c,e). B(c.e) A(c,e) = X(c,e) (see column 14, lines 34), where

A(c,e) is the Hermetian matrix. X(c,e) should be proportional to an identity matrix (refers to the attached the definitions of Hermitian matrix and Unitary matrix).

Conclusion

2. Any inquiry concerning this communication or earlier communications from the examiner should be directed to Shuwang Liu whose telephone number is (703) 308-9556.

If attempts to reach the examiner by telephone are unsuccessful, the examiner's supervisor, Stephen Chin, can be reached at (703) 305-4714.

Any response to this action should be mailed to:

Commissioner of Patents and Trademarks

Washington, D.C. 20231

or faxed to:

(703) 872-9306 (for Technology Center 2600 only)

Hand-delivered responses should be brought to Crystal Park II, 2121 Crystal Drive, Arlington, VA, Sixth Floor (Receptionist).

Any inquiry of a general nature or relating to the status of this application or proceeding should be directed to the Technology Center 2600 Customer Service Office whose telephone number is (703) 306-0377.

Shuwang Liu
Primary Examiner
Art Unit 2634

December 10, 2003

Hermitian Matrix

A square matrix is called Hermitian if it is self-adjoint. Therefore, a Hermitian matrix $A = (a_{ij})$ is defined as one for which

$$A = A^*, \tag{1}$$

where A^* denotes the adjoint matrix. This is equivalent to the condition

$$a_{ij} = \bar{a}_{ji}, \tag{2}$$

where $\bar{z}$ denotes the complex conjugate. As a result of this definition, the diagonal elements a_{ii} of a Hermitian matrix are real numbers (since $a_{ii} = \bar{a}_{ii}$), while other elements may be complex.

Examples of 2×2 Hermitian matrices include

$$\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}, \begin{bmatrix} 2 & -i \\ i & 1 \end{bmatrix} \tag{3}$$

and the Pauli matrices

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{4}$$

$$\sigma_2 = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} \tag{5}$$

$$\sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{6}$$

Examples of 3×3 Hermitian matrices include

$$\begin{bmatrix} -1 & 1-2i & 0 \\ 1+2i & 0 & -i \\ 0 & i & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1+i & 2i \\ 1-i & 5 & -3 \\ -2i & -3 & 0 \end{bmatrix}. \tag{7}$$

An integer or real matrix is Hermitian iff it is symmetric. A matrix m can be tested to see if it is Hermitian using the *Mathematica* function

```
HermitianQ[m_List?MatrixQ] := (m === Conjugate@Transpose@m)
```

Hermitian matrices have real eigenvalues whose eigenvectors form a unitary basis. For real matrices, Hermitian is the same as symmetric.

Any matrix C which is not Hermitian can be expressed as the sum of a Hermitian matrix and a skew Hermitian matrix using

$$C = \tfrac{1}{2}(C + C^*) + \tfrac{1}{2}(C - C^*). \quad (8)$$

Let U be a unitary matrix and A be a Hermitian matrix. Then the adjoint matrix of a similarity transformation is

$$\begin{aligned}(UAU^{-1})^* &= [(UA)(U^{-1})]^* = (U^{-1})^*(UA)^* \\ &= (U^*)^*(A^*U^*) = UAU^* = UAU^{-1}.\end{aligned} \quad (9)$$

The specific matrix

$$H(x,y,z) = \begin{bmatrix} z & x+iy \\ x-iy & -z \end{bmatrix} = xP_1 + yP_2 + zP_3, \quad (10)$$

where P_i are Pauli spin matrices, is sometimes called "the" Hermitian matrix.

SEE ALSO: Adjoint Matrix, Hermitian Operator, Hermitian Part, Normal Matrix, Pauli Spin Matrices, Skew Hermitian Matrix, Symmetric Matrix

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References

Arfken, G. "Hermitian Matrices, Unitary Matrices." §4.5 in *Mathematical Methods for Physicists, 3rd ed.* Orlando, FL: Academic Press, pp. 209-217, 1985.

Ayres, F. Jr. *Theory and Problems of Matrices.* New York: Schaum, pp. 13 and 117-118, 1962.

Unitary Matrix

This entry contributed by Todd Rowland

A square matrix U is a unitary matrix if

$$\mathsf{U}^* = \mathsf{U}^{-1}, \tag{1}$$

where U^* denotes the adjoint matrix and U^{-1} is the matrix inverse. For example,

$$\mathsf{A} = \begin{bmatrix} 2^{-1/2} & 2^{-1/2} & 0 \\ -2^{-1/2}i & 2^{-1/2}i & 0 \\ 0 & 0 & i \end{bmatrix} \tag{2}$$

is a unitary matrix. A matrix m can be tested to see if it is unitary using the *Mathematica* function

```
UnitaryQ[m_List?MatrixQ] :=
  (Conjugate@Transpose@m.m == IdentityMatrix@Length@m)
```

The definition of a unitary matrix guarantees that

$$\mathsf{U}^*\mathsf{U} = \mathsf{I}, \tag{3}$$

where I is the identity matrix. In particular, a unitary matrix is always invertible, and $U^{-1} = U^*$. Note that transpose is a much simpler computation than inverse. Unitary matrices leave the length of a complex vector unchanged. A similarity transformation of a Hermitian matrix with a unitary matrix gives

$$\begin{aligned} (uau^{-1})^* &= [(ua)(u^{-1})]^* = (u^{-1})^*(ua)^* = (u^*)^*(a^*u^*) \\ &= uau^* = uau^{-1}. \end{aligned} \tag{4}$$

Unitary matrices are normal matrices. If M is a unitary matrix, then the permanent

$$|\mathrm{perm}(\mathsf{M})| \leq 1 \tag{5}$$

(Minc 1978, p. 25, Vardi 1991).

For real matrices, unitary is the same as orthogonal. In fact, there are some similarities between orthogonal matrices and unitary matrices. The rows of a unitary matrix are a unitary basis. That is, each row has length one, and their Hermitian inner product is zero. Similarly, the columns are also a unitary basis. In fact, given any unitary basis, the matrix whose rows are that basis is a unitary matrix. It is automatically the case that the columns are another unitary basis.

The unitary matrices are precisely those matrices which preserve the Hermitian inner product

$$\langle v, w\rangle = \langle Uv, Uw\rangle. \tag{6}$$

Also, the norm of the determinant of U is $|\det U| = 1$. Unlike the orthogonal matrices, the unitary matrices are connected. If $\det U = 1$ then U is a special unitary matrix.

The product of two unitary matrices is another unitary matrix. The inverse of a unitary matrix is another unitary matrix, and identity matrices are unitary. Hence the set of unitary matrices form a group, called the unitary group.

SEE ALSO: Adjoint Matrix, Antihermitian Matrix, Clifford Algebra, Group Representation, Hermitian Inner Product, Hermitian Matrix, Normal Matrix, Orthogonal Group, Permanent, Special Unitary Matrix, Spin Group, Symmetric Matrix, Unimodular Matrix, Unit Matrix, Unitary Group

References

Minc, H. §3.1 in *Permanents.* Reading, MA: Addison-Wesley, 1978.

Vardi, I. *Computational Recreations in Mathematica.* Reading, MA: Addison-Wesley, 1991.

Eric W. Weisstein